# Core Mathematics C3 For Edexcel
# Advanced Level

## Paper J
## Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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**1.** Simplify

$$\frac{4x^2 - 25}{x^2 + x} \div \frac{2x^2 - x - 10}{x^2 + 3x + 2}.$$ **(6)**

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**2.** (*a*) Given $x = \tan y$, find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ and hence find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$. **(4)**

(*b*) Show that $(1 + x^2)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 2x\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$. **(3)**

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**3.** The $n^{\text{th}}$ term of an arithmetic progression is $\ln(pq^{n-1})$ where $p$ and $q$ are positive integers.

(*a*) What is the first term of the sequence? **(1)**

(*b*) Show that the common difference is $\ln q$. **(2)**

(*c*) Find, in terms of $\ln p$, $\ln q$ and $n$, the sum of the first $n$ terms of the series formed by the terms of the sequence. **(3)**

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**4.** The root of the equation $\mathrm{f}(x) = 0$, where

$$\mathrm{f}(x) = 2x + \ln 3x - 5$$

is to be estimated by using an iterative formula.

(*a*) Show that the root $\alpha$, such that $\mathrm{f}(\alpha) = 0$, lies in the interval [1, 2]. **(2)**

(*b*) Show that $\mathrm{f}(x) = 0$ can be rewritten as

$$x = \frac{1}{2}(5 - \ln 3x).$$ **(2)**

(*c*) Use the iteration

$$x_{n+1} = \frac{1}{2}(5 - \ln 3x_n) \quad \text{with} \quad x_0 = 1.5,$$

to obtain the values of $x_1, x_2, x_3$ and $x_4$. **(2)**

(*d*) Give the value of $\alpha$ correct to 3 decimal places. **(1)**

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**5.** (*a*) Given that

$$\cos(2x - 60) = 2\sin(2x + 30),$$

prove that $\tan 2x = -\dfrac{1}{\sqrt{3}}$. **(5)**

(*b*) Using the result from part (*a*), find two values of $x$, $0 < x < 180°$, which satisfy the equation

$$2\sin(2x + 30) - \cos(2x - 60) = 0.$$ **(3)**

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**6.** (*a*) On the same axes sketch the graphs of $C_1$, $y = \mathrm{e}^{\frac{1}{2}x}$, and $C_2$, $y = \mathrm{e}^{-2x}$.

The graphs intersect at the point $A$. **(4)**

(*b*) State the coordinates of $A$. **(1)**

(*c*) Prove that the tangent at point $A$ to the curve $C_1$ is the normal to the curve $C_2$ at the same point. **(5)**

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**7.** Differentiate with respect to $x$,

(*a*) $(3x + 1)^7$, **(3)**

(*b*) $\ln\sqrt{(4x + 1)}$, **(3)**

(*c*) $\cos 7x$. **(3)**

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**8.** The function f is defined by

$$\mathrm{f}\colon x \mapsto |2x - 1| - 4, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph of $y = \mathrm{f}(x)$. **(2)**

(*b*) Solve the equation $\mathrm{f}(x) = 3$. **(3)**

The function g is defined by

$$\mathrm{g}\colon x \mapsto x^2 - 8x + 17, \quad x \geq 0.$$

(*c*) Find the range of g. **(3)**

(*d*) Find $\mathrm{gf}(3)$. **(2)**

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**9.** (*a*) (i) Express

$$9\cos\theta - 40\sin\theta \qquad \text{in the form}$$

$$R\cos(\theta + \alpha) \quad \text{where} \quad R > 0 \quad \text{and} \quad 0 < \alpha < \frac{\pi}{2}.$$ **(4)**

(ii) Hence solve the equation

$$9\cos\theta - 40\sin\theta = 6,$$

for $0 < \theta < \frac{\pi}{2}$, giving your answer to 2 decimal places. **(3)**

(*b*) Solve the equation

$$13 + 10\cot\theta = 3\tan\theta,$$

for $0 < \theta < \frac{\pi}{2}$, giving your answer to 2 decimal places. **(5)**

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**END** **TOTAL 75 MARKS**